A Modular Site of Reconstruction Regimes: Twelve Representations of Quantum Gravity and the Weakest-Link Calculus that Composes Them

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Abstract

This paper collects twelve companion studies into one structure without collapsing them into one theory. The organizing thesis is that quantum gravity, in the sense used across this program, is not the quantization of matter fields on a fixed background. It is the reconstruction of spacetime, matter, causality, and measurement as *outputs* of invariant representation structures. We treat each of the twelve regimes—classical Lorentzian geometry, BV–BRST gauge cohomology, spin networks, spin foams, tensor-network holography and quantum error correction, positive geometries, the double copy, celestial conformal data, asymptotic safety, factorization algebras, spectral (noncommutative) geometry, and quantum causal structures—as a separate representation of the same reconstruction question, referenced here as Parts I through XII. Each carries an epistemic status drawn from a three-valued warrant lattice $\mathsf{W} = \{S < H < P\}$ (standard, heuristic, speculative). The central claim is made precise as a calculus rather than a slogan: regimes combine by a composition rule whose governing warrant is the join (the worst) of its inputs, so a chain of reasoning is never more secure than its weakest link. We prove the elementary but load-bearing facts about this calculus (it is a bounded, idempotent, commutative monoid with unit S and absorbing element P; composition is monotone; speculative inputs are never silently upgraded), give a unified representation-stack and realization-pipeline framework in which all twelve regimes are instances of "physical content is a groupoid, not a coarse quotient," and work through explicit compositions that show emergent structure appearing at each level while the composite status is tracked honestly. We catalogue the structural recurrences that thread several regimes at once (labels-as-geometry, residue/coaction trees, gauge redundancy versus code redundancy, local-to-global descent, causal order, emergent metric) and, equally important, we state the identifications that are *not* licensed: celestial and anti–de Sitter holography are kept distinct, and the spin-foam continuum limit and the asymptotic-safety fixed point are left as an open, unbridged comparison. We close with a limitations section and a consolidated open-problem queue. The result is a map of a modular site, not a master equation.

Contents

1 Introduction

1.1 The reconstruction thesis

The governing perspective of this program is stated once and then applied without exception:

> Quantum gravity is not primarily the quantization of material objects in spacetime. It is the reconstruction of spacetime, matter, causality, and measurement from invariant representation structures.

Read literally, this reorders the usual dependency. In the standard picture one starts with a spacetime manifold, places fields on it, and quantizes. Here the manifold, the causal relations on it, the matter content, and the notion of a measurement outcome are treated as things to be produced. The inputs are representation-theoretic and algebraic: a graph labelled by group representations, a spectral triple, a positive geometry with a canonical form, a process matrix, a fixed point of a renormalization-group flow. The outputs are the familiar geometric objects, recovered as theorems, limits, or conjectures with explicit caveats. This is a program about *what reconstructs what*, and about how honestly each reconstruction can be claimed.

The twelve companion papers each execute one reconstruction. This synthesis does not add a thirteenth. Its job is to state the relations among the twelve precisely enough that a reader can see both what they share and where they must be kept apart.

1.2 Modular, not unified

There is a strong temptation, when twelve frameworks all speak about the same physical target, to announce that they are "really" one framework in disguise. We resist this. The design promise inherited from the underlying representation library is explicit: the library "treats quantum gravity as a modular site of partially overlapping representation regimes," and it "does not collapse spin foams, holography, amplitudes, asymptotic safety, celestial holography, and categorical formalization into one monolithic theory." The present paper is written to honor that promise, and the honoring is done with a specific device.

Each regime carries an epistemic status: a warrant in the ordered set $\mathsf{W} = \{S < H < P\}$, where S marks a standard mathematical or mathematical-physics correspondence, H marks a strong heuristic physical representation, and P marks a speculative ontological extension. Combining regimes never improves a warrant. The governing status of a chain of reasoning is the join (the maximum, the worst) of the statuses of the links used. This is a theorem about a very small algebraic structure, and it is the whole content of the word "modular" as we use it. It says that no amount of careful work in a mature regime can launder a speculative link that the argument also depends on. A single claim that leans on all twelve regimes at once inherits the worst label present, which for the current census is P.

We therefore do not write "regime A subsumes regime B." We write "a claim built from entries of A and B has the weakest-link status of the entries actually used," and we track that status through every worked example.

1.3 What this paper contains

Section 2 sets up the unified framework: the representation stack and realization pipeline that all twelve regimes instantiate, and the S/H/P warrant calculus, with the elementary structural results that make "weakest link" a proved statement rather than a metaphor. Section 3 presents the twelve regimes as Parts I–XII, each with a short structural summary, its core invariant, its status, and a pointer to the corresponding companion paper. Section 4 organizes the regimes into a reconstruction ladder and shows how composition produces structure that is present in neither factor alone. Section 5 works through explicit weakest-link compositions. Section 6 catalogues the cross-cutting structural recurrences. Section 7 states the non-identifications that must be respected. Section 8 gives the module map from the twelve regimes to the roadmap modules QG-I through QG-X. Section 9 isolates the emergent-property theme. Section 10 lists limitations and the open-problem queue. Section 11 concludes.

Throughout, status tags appear inline: **[S]**, **[H]**, **[P]**, and the composites **[S/H]** and **[H/P]**. They are not decoration. They are the bookkeeping that keeps the synthesis honest.

2 The unified framework

The twelve regimes look different on the surface. A Lorentzian manifold, a factorization algebra, and a process matrix are not obviously the same kind of object. What they share is a single abstract shape: each is a rule that assigns, to a piece of physical input data, a groupoid of representations whose isomorphisms record the redundancy of description, together with a realization step that turns the representation into observables. This section makes that shape precise and then builds the warrant calculus on top of it.

2.1 The representation stack

Definition 2.1 (Representation entry)**.** A *representation entry* is a tuple $E = (M, P, \tau, \sigma)$ consisting of a mathematical object M, a physical target P, a translation datum τ relating them, and an epistemic status $\sigma \in \mathsf{Status}$ (theorem 2.5). The entry asserts that M represents P via τ, at warrant σ.

Definition 2.2 (Realization pipeline)**.** A *realization pipeline* on an object M is the composite

$$M \xrightarrow{\Phi} \Phi(M) \xrightarrow{\mathrm{Real}_\alpha} \mathrm{Real}_\alpha(\Phi(M)) \longrightarrow \text{physical representation} \xrightarrow{\mathrm{Obs}} \mathrm{Obs},$$

so that the observable content is $\mathrm{Obs}_\alpha(M) = \mathrm{Obs}\big(\mathrm{Real}_\alpha(\Phi(M))\big)$. Here Φ extracts the invariant structure, Real_α realizes it in a chosen physical scheme α, and Obs reads off measurable quantities.

The key structural fact, which recurs in every regime, is that the intermediate representation is a groupoid rather than a set. Two representatives related by an isomorphism carry the same observable content, and the isomorphisms themselves carry information (gauge redundancy, stabilizers, isotropy) that a naive quotient would discard.

Definition 2.3 (Representation prestack and stack)**.** A *representation prestack* is a pseudofunctor $\mathrm{Rep}\colon \mathrm{Dom}^{\mathrm{op}} \to \mathbf{Gpd}$ from a category of physical domains to groupoids. It is a *representation stack* for a Grothendieck topology on Dom when compatible local entries glue to a global entry, uniquely up to coherent isomorphism. Gluing is by equivalence, not by equality, so automorphism data is retained.

Principle 2.4 (Physical content is a groupoid, not a coarse quotient). In every regime the physically meaningful datum is an object of a groupoid $[X/G]$ together with its automorphism group, and for an action groupoid one has

$$\mathrm{Aut}_{[X/G]}(x) \;\cong\; \mathrm{Stab}_G(x).$$

Passing to the coarse orbit set X/G forgets $\mathrm{Stab}_G(x)$ and with it the symmetry, degeneracy, and anomaly data that later reconstructions must reproduce.

Theorem 2.4 is not a theorem in the strict sense; it is the shared shape that the twelve regimes each realize with their own technical content. In Part I it is the identification of the isotropy of the diffeomorphism action with the Killing group. In Part II it is the ghost-for-ghost tower of the BV–BRST complex. In Part III it is the intertwiner space attached to a spin-network vertex. We return to this in section 6.

2.2 The warrant calculus

We now build the epistemic bookkeeping. Let $\mathsf{W} = \{S, H, P\}$ be the totally ordered set

$$S \;<\; H \;<\; P,$$

with S the least element. Read S as "established," H as "heuristic," P as "speculative." Larger means weaker. Write $\vee$ for the binary maximum (join) on W.

Definition 2.5 (Status)**.** A *status* is a nonempty up-closed-free label built from warrants, rendered S, H, P, or a composite such as S/H or H/P that records a range of warrants over the parts of a construction. To each status σ we assign its *governing warrant* $\mathrm{w}(\sigma) \in \mathsf{W}$, the join of the warrants occurring in σ. Thus $\mathrm{w}(S/H) = H$, $\mathrm{w}(H/P) = P$, and $\mathrm{w}(S) = S$.

The governing warrant is what composition tracks. It is the worst thing that can happen along the construction.

Definition 2.6 (Composition of statuses)**.** The composite of statuses $\sigma_1, \ldots, \sigma_n$ is the status whose governing warrant is

$$\mathrm{w}\big(\sigma_1 \vee \cdots \vee \sigma_n\big) \;=\; \mathrm{w}(\sigma_1) \vee \cdots \vee \mathrm{w}(\sigma_n) \;=\; \max_i \mathrm{w}(\sigma_i),$$

rendered as the single warrant $\max_i \mathrm{w}(\sigma_i)$.

This reproduces the executable calculus of the underlying library, in which, for example, $\mathrm{compose}(S, S/H) = H$ and $\mathrm{compose}(S, H, P) = P$. The next three results are elementary. We state and prove them because they are exactly the formal content of the modular claim, and because the formalization roadmap (open-problem 7 in section 10) asks for machine-checkable versions.

Proposition 2.7 (The warrant monoid)**.** $(\mathsf{W}, \vee, S)$ *is a commutative, idempotent monoid:* $\vee$ *is associative and commutative,* $x \vee x = x$ *for all* x*, and* $S \vee x = x$ *for all* x*. Moreover* P *is absorbing,* $P \vee x = P$*, and* $\vee$ *is monotone in each argument.*

Proof. W is a finite totally ordered set, so it is a lattice, and $\vee$ is its join. Join on any lattice is associative, commutative, and idempotent. The least element S satisfies $S \vee x = x$, so S is the identity; the greatest element P satisfies $P \vee x = P$, so P is absorbing. Monotonicity is immediate from $x \le x' \Rightarrow x \vee y \le x' \vee y$, which holds in any lattice. □

Theorem 2.8 (Weakest link)**.** *Let a claim be assembled by composing entries with statuses* $\sigma_1, \ldots, \sigma_n$*. Then the governing warrant of the assembled claim equals* $\max_i \mathrm{w}(\sigma_i)$*. In particular:*

(i) the assembled claim is at least as weak as each of its links, $\mathrm{w}(\mathit{claim}) \ge \mathrm{w}(\sigma_i)$ *for every* i*;*

(ii) composition never strengthens: no chain of compositions produces a warrant strictly below $\min_i \mathrm{w}(\sigma_i)$*, and in fact the result equals the maximum, so the warrant only ever moves toward* P*;*

(iii) if some link is speculative, $\mathrm{w}(\sigma_k) = P$*, then the whole claim is speculative.*

Proof. By theorem 2.6 the governing warrant of the composite is $\bigvee_i \mathrm{w}(\sigma_i) = \max_i \mathrm{w}(\sigma_i)$, using theorem 2.7 to reassociate the join arbitrarily. Statement (i) is $x_j \le \max_i x_i$. Statement (ii): the maximum of a finite set lies between its minimum and its maximum, and by the displayed equality the composite equals the maximum, so it is $\ge$ every input and cannot fall below any of them. Statement (iii): if $\mathrm{w}(\sigma_k) = P$ then by absorption $\max_i \mathrm{w}(\sigma_i) = P$. □

Corollary 2.9 (No silent upgrade)**.** *There is no status σ and no composition that maps a governing warrant P to a governing warrant strictly below P. Speculative entries are never silently upgraded by being placed in a chain with better-warranted entries.*

Proof. Immediate from theorem 2.8(iii): any composite containing a P link has governing warrant P. □

Theorem 2.9 is the sentence "speculative entries are never silently upgraded" turned into a proof obligation that the calculus discharges by construction. It is the reason the composition of all twelve regimes lands at P (section 5), and the reason we never present such a composition as better than speculative.

Remark 2.10 (What the calculus does not do). Theorem 2.8 bounds a composite status from above by the worst link. It does *not* assert that any bridge between two regimes exists. A composite warrant of H for a claim linking regime A and regime B means only "neither endpoint exceeds heuristic status, so nothing built from them can either." It is silent on whether the bridge $A \to B$ has ever been constructed. Several of the compositions in section 5 are of exactly this kind: the calculus assigns a bound, while the bridge itself remains an open problem.

2.3 Regimes as objects of the framework

We package a regime uniformly so that the twelve Parts can be compared.

Definition 2.11 (Regime)**.** A *regime* is a tuple $R = (\mathrm{Dom}_R, \mathrm{Rep}_R, \iota_R, \sigma_R)$ where Dom_R is a domain category of physical inputs, $\mathrm{Rep}_R \colon \mathrm{Dom}_R^{\mathrm{op}} \to \mathbf{Gpd}$ is a representation prestack (theorem 2.3), ι_R names the *core invariant*—the equivalence-invariant datum through which Rep_R factors and by which the regime's physical content is defined—and $\sigma_R \in \mathsf{Status}$ is the regime's status. A *bridge* $B \colon R \to R'$ is a partial functor from a subdomain of Rep_R to $\mathrm{Rep}_{R'}$ compatible with realization on the overlap; it carries its own status σ_B, and a chain of bridges composes its status by theorem 2.6.

With this vocabulary the whole paper is: list the twelve regimes and their core invariants (section 3); identify the bridges (sections 4 and 5); compute composite statuses (section 5); and mark the pairs where no bridge is licensed (section 7).

3 The twelve regimes

Each subsection below is one Part. We give the regime's role in the reconstruction program, its core invariant ι_R in the sense of theorem 2.11, its status σ_R, and a one-paragraph structural summary. The companion papers carry the full development; we cite them as Parts I–XII.

Part I — Lorentzian geometry: the classical target

Role. The classical general-relativity regime, and the semiclassical shadow that every deeper reconstruction must reproduce [1]. **Core invariant.** Diffeomorphism equivalence: a

Lorentzian metric (M, g) up to $\mathrm{Diff}(M)$, packaged as the moduli *stack* $\mathrm{Sol(Ein)}/\!\!/\mathrm{Diff}(M)$ of Einstein solutions, whose automorphism group at g is the isometry group, $\mathrm{Aut}_{[\mathrm{Sol}/\mathrm{Diff}]}([g]) \cong \mathrm{Isom}(M, g)$. **Status.** **[S/H]**.

The physically meaningful object is not a point in the coarse orbit set $\mathrm{Sol(Ein)}/\mathrm{Diff}(M)$ but an object of the action groupoid, together with its Killing isotropy. The isotropy of the $\mathrm{Diff}(M)$-action at a metric g is the isometry group $\mathrm{Isom}(M, g)$. On the space of *Riemannian* metrics the Ebin slice theorem makes this precise: the action is proper, isometry groups are compact, and a genuine slice exists. The gravitational setting is Lorentzian, where the $\mathrm{Diff}(M)$-action on metrics is not proper and isometry groups are generally noncompact, so the classical slice theorem does not apply directly; one passes instead to the initial-value formulation, where the slice theorem on the Riemannian metrics of a Cauchy slice and the Fischer–Marsden–Moncrief linearization-stability results show the coarse quotient acquires conical singularities exactly at solutions carrying Killing fields, where the stack stays smooth. The failure of a naive Lorentzian slice theorem is itself part of why the stack $\mathrm{Sol(Ein)}/\!\!/\mathrm{Diff}(M)$, and not the coarse quotient, is the right carrier. Part I is the concrete instance of theorem 2.4 in its most classical form, and it fixes the target that Parts III, IV, V, VIII, IX, XI, and XII must recover in a limit.

Part II — BV–BRST and derived geometry: the gauge layer

Role. Gauge redundancy and constraints resolved homologically; observables as cohomology [2]. **Core invariant.** Observables are the degree-zero BV–BRST cohomology $H^0(Q_{\mathrm{BV}})$ of the field–antifield–ghost complex, equivalently H^0 of the structure sheaf of the derived critical locus $\mathrm{dCrit}(S) = X \times^h_{T^*X} X$—the derived intersection, inside T^*X, of the graph of dS with the zero section, which by the PTVV theorem carries a canonical (-1)-shifted symplectic structure and locally models the BV space $T^*[-1]X$—with its BV differential, invariant under both classical gauge transformations and the choice of gauge fixing. **Status.** **[S/H]**.

BV–BRST is the technology that turns theorem 2.4 into a computation for perturbative field theory. Ghosts encode infinitesimal gauge directions; antifields encode the equations of motion and their reducibility; the derived critical locus is the homotopical model of $\mathrm{Sol(Ein)}/\!\!/\mathrm{Diff}$ at the perturbative level, with the ghost-for-ghost tower recording exactly the isotropy that the coarse quotient of Part I would erase. The regime's own limitation is that its nonperturbative global completion is model-dependent, which is the weak link whenever Part II is chained to a nonperturbative regime.

Part III — Spin networks: quantum spatial geometry

Role. Quantum geometry of a spatial slice, built from $SU(2)$ representation theory [3]. **Core invariant.** The kinematical Hilbert space $\mathcal{H}_{\mathrm{kin}}$ with its spin-network basis—graphs Γ with edges labelled by spins j_e and vertices by intertwiners ι_v—together with the discrete spectra of the area and volume operators, invariant under $SU(2)$ gauge transformations at vertices and under diffeomorphisms of the graph embedding. **Status.** **[S/H]**.

Geometry appears here as representation data: an area eigenvalue is a sum of Casimir numbers $\sqrt{j_e(j_e+1)}$ over edges piercing a surface, and a volume eigenvalue is built from the intertwiner combinatorics at a vertex. The Gauss-constraint quotient that produces the intertwiner space is once again theorem 2.4: the gauge-invariant content at a vertex is a groupoid quotient, not a set. The heuristic component of the status enters at the bridge to Part I: the claim that a coherent or large-spin limit reproduces a specific classical three-geometry is only heuristically established.

Part IV — Spin foams: covariant amplitudes

Role. Covariant transition amplitudes between spin-network boundary states [4]. **Core invariant.** The amplitude $Z(\mathcal{F})\colon \mathcal{H}_{\Gamma_{\mathrm{in}}} \to \mathcal{H}_{\Gamma_{\mathrm{out}}}$ associated to a two-complex $\mathcal{F}$ with face and edge labels, as a state sum over representations, together with its behavior under refinement of $\mathcal{F}$. **Status.** **[H]**.

The vertex amplitude assigns a number to a labelled cell, and the full amplitude is a sum over colorings. The semiclassical asymptotics of the EPRL/Barrett–Crane vertex reproduce a Regge action, which is the sense in which curved geometry is recovered from purely combinatorial $\{6j\}/\{15j\}$ data. The status is bare heuristic because the refinement/continuum limit—whether and how the amplitude stabilizes as the discretization is refined—is not settled. Part IV is the first regime whose weak link drags composite chains down to H.

Part V — Tensor networks and QEC: holographic emergence

Role. Emergence of a bulk geometry from boundary entanglement, modelled as quantum error correction [5]. **Core invariant.** The isometric encoding $\mathcal{H}_{\mathrm{bulk}} \hookrightarrow \mathcal{H}_{\mathrm{bdy}}$ together with its complementary-recovery and entanglement-wedge structure: a bulk operator is an equivalence class of boundary representatives, and any boundary region whose entanglement wedge contains the bulk point can reconstruct it. **Status.** **[H]** for the holographic claim, **[S/H]** for the toy-model (HaPPY-code, stabilizer) mathematics.

The slogan is "many boundary representatives, one bulk operator." The decomposition $\mathcal{H}_{\mathrm{bdy}} \simeq (\mathcal{H}_{\log} \otimes \mathcal{H}_{\mathrm{gauge}}) \oplus \mathcal{H}_{\mathrm{err}}$ is exact for the stabilizer toy models, which is why the code mathematics is S/H, while the claim that this is how semiclassical bulk gravity actually emerges beyond anti–de Sitter examples remains open, which is why the dominant status is H. This regime is the sharpest realization of the "reconstruction of measurement" half of the thesis, and its relation to Part II's gauge redundancy is a named open problem.

Part VI — Positive geometries: amplitudes as canonical forms

Role. Scattering amplitudes reconstructed from boundary and residue data, with no Lagrangian or path integral in the definition [6]. **Core invariant.** The canonical form Ω_X of a positive geometry $X_{\geq 0}$: the unique top form with logarithmic singularities on, and unit residues along, the boundary stratification of $X_{\geq 0}$; the recursive residue tree *is* the amplitude. **Status.** **[S/H]** for the canonical-form construction, degrading to **[H]** once the conjectural bridge to motivic coactions is invoked.

The amplituhedron and its relatives realize a scattering amplitude as the canonical form of a region cut out by positivity conditions. The recursive structure of residues along boundaries mirrors the physical factorization of amplitudes on cuts. The heuristic component enters through open-problem 1, the conjecture that the residue tree of the geometry coincides with the coaction tree of the motivic period attached to the amplitude.

Part VII — The double copy: gravity from gauge squared

Role. Perturbative gravity amplitudes constructed from two copies of gauge-theory data [7]. **Core invariant.** Color–kinematics duality: kinematic numerators n_i that satisfy the same Jacobi and antisymmetry relations as the color factors c_i, so that the replacement $c_i \mapsto \tilde{n}_i$ sends a gauge amplitude to a gravity amplitude, controlled at tree level by the KLT kernel. **Status.** **[S/H]**.

"Gravity is the square of gauge theory" is, at the level of perturbative amplitudes, a precise statement once dual numerators exist: the double copy is well-defined exactly on the locus where color–kinematics-satisfying numerators can be found. Part VII shares module QG-IV with Part VI, and the two combine (with the coaction conjecture) into the open-problem-1 test case of section 5.

Part VIII — Celestial CFT: flat-space boundary data

Role. The flat-space S-matrix rewritten as conformal data on the celestial sphere [8]. **Core invariant.** Soft-theorem and asymptotic-symmetry Ward identities: the constraints of (super)translation and (super)rotation symmetry, realized as celestial conformal and current-algebra Ward identities on Mellin-transformed amplitudes, exactly as ordinary CFT Ward identities constrain correlators. **Status.** **[H]**.

A Mellin transform in the external energies turns four-dimensional scattering amplitudes into objects that transform as two-dimensional conformal correlators. Soft theorems become Ward identities for the BMS asymptotic symmetries. This is a change of basis on the same S-matrix data, not a replacement of it, which is central to the non-identification of section 7: celestial holography is not anti–de Sitter holography in different coordinates.

Part IX — Asymptotic safety: the RG completion

Role. An ultraviolet completion of gravity as a nontrivial renormalization-group fixed point [9]. **Core invariant.** The non-Gaussian fixed point g_* in theory space together with its critical exponents and universality class: an invariant of the RG flow that, if it is truncation-independent, characterizes the ultraviolet completion regardless of which operator basis is used to compute it. **Status.** **[H]**.

Asymptotic safety proposes that the gravitational couplings run to a fixed point with finitely many relevant directions, giving a predictive continuum theory. The core invariant is the fixed point and its critical data; the persistent caveat is that the evidence comes from truncations of theory space, and truncation-independence is not established. Part IX supplies the "does it have a good continuum limit?" question in its renormalization-group form, structurally parallel to Part IV's refinement question but technically unrelated to it (section 7).

Part X — Factorization algebras: local-to-global observables

Role. The local-to-global assembly of quantum observables [10]. **Core invariant.** Factorization: $\mathrm{Obs}(U \cup V)$ is built from $\mathrm{Obs}(U)$, $\mathrm{Obs}(V)$, and their overlap data by a colimit/composition, so that observables on a larger region are *composed* from those on smaller regions rather than restricted from a global datum. **Status. [S/H]**.

Factorization algebras (equivalently, the Costello–Gwilliam packaging of algebraic quantum field theory) formalize how measurements in adjacent regions combine. They share module QG-II with Part II: the BV–BRST observables of a classical theory assemble into a factorization algebra, and the obstruction to quantizing it is a cohomological existence statement. Part X globalizes Part II's descent logic, and it is the natural home for the local-to-global recurrence of section 6.

Part XI — Noncommutative geometry: spectral reconstruction

Role. Metric geometry reconstructed from spectral data, with no manifold assumed in advance [11]. **Core invariant.** The spectrum of the Dirac operator D, and the algebraic structure of the commutators $[D, a]$ for a in the algebra $\mathcal{A}$: geometry is an invariant of the spectral triple $(\mathcal{A}, \mathcal{H}, D)$ up to unitary equivalence, with metric distance recovered by the Connes distance formula and dynamics by the spectral action. **Status. [H]**.

Connes's reconstruction theorem recovers a Riemannian spin manifold from a commutative spectral triple satisfying suitable axioms, so "geometry is a theorem about an algebra." The heuristic status reflects two gaps: a fully general Lorentzian (causal) analogue of the reconstruction theorem is not available, and the phenomenological content of the spectral action depends on cutoff-scale matching. This regime maps to the proposed module QG-IX (section 8).

Part XII — Quantum causal structures: indefinite causal order

Role. Causal order itself treated as the primitive datum, classically as a locally finite poset and quantum-mechanically as a process matrix [12]. **Core invariant.** The order relation $(\mathcal{C}, \prec)$ of a causal set, or the process matrix W encoding all statistics consistent with local quantum mechanics without presupposing a fixed background order; geometry (causal sets) and operationally testable causal indefiniteness (process matrices) are read off from this primitive. **Status. [H/P]** (proposed; the process-matrix content has no prior dictionary entry).

Causal-set theory reconstructs a Lorentzian manifold from order plus counting (the Hauptvermutung, still conjectural). Process matrices describe correlations, such as those of the quantum switch, that violate causal inequalities and so certify indefinite causal order operationally. The speculative half of the status marks the step from "causally nonseparable processes are mathematically consistent" to "quantum gravity dynamically produces indefinite causal order," which is not established. This regime maps to the proposed module QG-X, and it is the source of the P that caps any all-twelve composition (section 5).

Table 1 collects the statuses. It is the input to every composition in the rest of the paper.

Part	Regime	Core invariant ι_R	σ_R
I	Lorentzian geometry	diffeomorphism equivalence; Isom isotropy	**[S/H]**
II	BV–BRST / derived	$H^0(Q_{\mathrm{BV}})$; derived critical locus	**[S/H]**
III	spin networks	$\mathcal{H}_{\mathrm{kin}}$; area/volume spectra	**[S/H]**
IV	spin foams	state-sum amplitude $Z(\mathcal{F})$; refinement	**[H]**
V	tensor networks / QEC	isometric encoding; entanglement wedge	**[H]**
VI	positive geometries	canonical form Ω_X; residue tree	**[S/H]**
VII	double copy	color–kinematics duality; KLT	**[S/H]**
VIII	celestial CFT	asymptotic-symmetry Ward identities	**[H]**
IX	asymptotic safety	fixed point g_*; critical exponents	**[H]**
X	factorization algebras	local-to-global gluing by composition	**[S/H]**
XI	noncommutative geometry	spectrum of D; $(\mathcal{A}, \mathcal{H}, D)$	**[H]**
XII	quantum causal structures	poset $(\mathcal{C}, \prec)$ / process matrix W	**[H/P]**

Table 1: The twelve regimes, their core invariants, and their statuses. Composite statuses everywhere in this paper are computed from this column by theorem 2.6.

4 The reconstruction ladder

The twelve regimes are not a flat list. They arrange into levels, each taking the output of the previous as input, and each producing structure that was not visible below. This section describes the ladder and the composition arrows on it; the next section computes the associated statuses.

4.1 Levels

Level 0 — classical target (Part I). A Lorentzian solution up to diffeomorphism, with isotropy retained. Everything above must reproduce this in a limit.

Level 1 — gauge and observables (Parts II, X). The redundancy of Level 0 is resolved into a cohomological observable algebra (Part II), and that algebra is assembled from local pieces by factorization (Part X). This is the descent layer.

Level 2 — quantum spatial geometry (Part III). The spatial slice of Level 0 is replaced by representation data: a spin network whose labels carry area and volume.

Level 3 — dynamics and amplitudes (Parts IV, VI, VII). Transitions between Level-2 states (spin foams, Part IV) and perturbative amplitudes as canonical forms (Part VI) and as gauge-squared data (Part VII). This is where a process, not just a state, is represented.

Level 4 — boundary and holographic emergence (Parts V, VIII). The bulk of Levels 0–3 is re-encoded on a boundary: as an error-correcting code (Part V, anti–de Sitter) or as celestial conformal data (Part VIII, flat). These are two *distinct* boundary reconstructions, not one.

Level 5 — ultraviolet completion (Parts IV, IX). Whether the constructions have a good short-distance limit, asked as a refinement question (Part IV) and as a fixed-point question (Part IX). These two are, at present, unbridged.

Transverse reconstructions (Parts XI, XII). Two regimes reconstruct Level-0 geometry from data that is neither field-theoretic nor holographic: spectral data (Part XI) and causal order (Part XII). They cut across the ladder rather than sitting on one rung.

4.2 Bridges as composition arrows

Each solid arrow in fig. 1 is a bridge in the sense of theorem 2.11, with a status of its own. Crucially, the bridge's status can be worse than either endpoint's, because the bridging claim may be less secure than the regimes it connects. Section 5 makes this explicit: the composition of two S/H regimes can be H when the bridge between them is only heuristically established.

The dashed arrows (Parts XI, XII) are drawn to Level 0 because their reconstruction targets are directly the classical metric and causal structure, not the intermediate field-theoretic layers. This placement is also a warning: the dashed arrows do *not* connect to Parts III/IV (a spectral triple is not a spin network, and a causal set is not a spin foam), a non-identification we state in section 7.

5 Worked weakest-link compositions

We now compute. Each composition below names the regimes used, states the bridging claim, applies theorem 2.6, and reports the honest composite status. In several cases the point of the computation is precisely that the calculus bounds the status while the bridge itself is unproven (theorem 2.10).

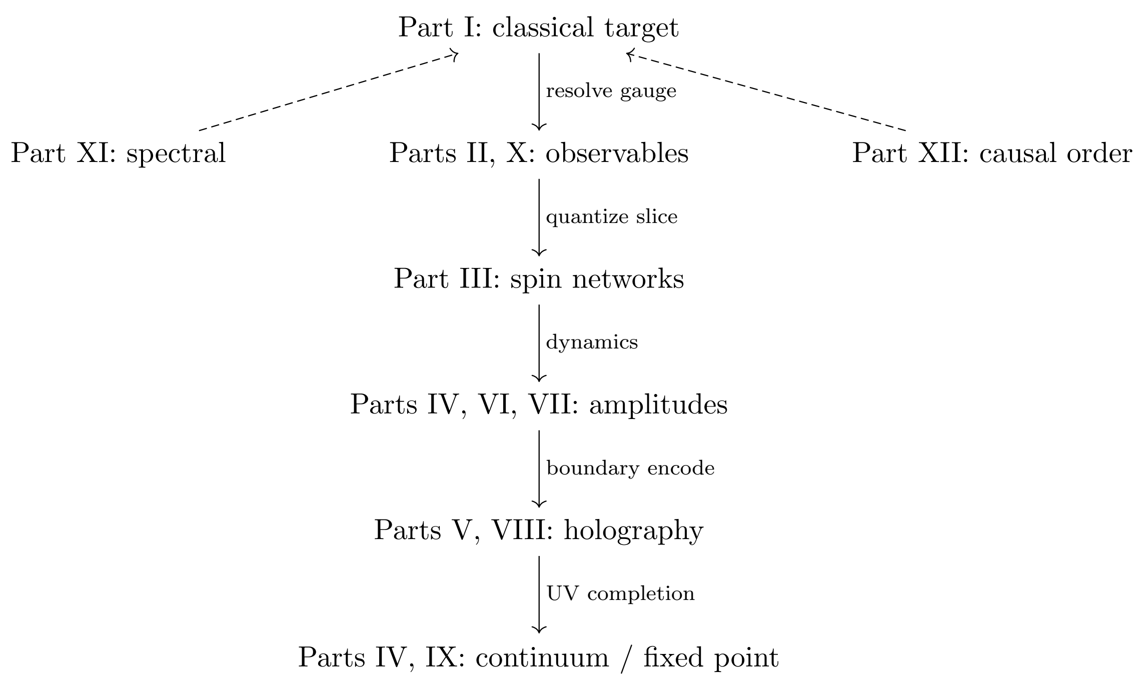

Figure 1: The reconstruction ladder. Solid arrows are composition bridges within the ladder; dashed arrows are the transverse reconstructions of Parts XI and XII, which target Level-0 geometry directly. Every arrow carries a status by theorem 2.11, and the status of a path is the join of its arrows by theorem 2.8.

5.1 Part I with Part III: a semiclassical slice from spin networks

Claim. A large-spin or coherent-state limit of a spin network reproduces a specific classical three-geometry, so that the Part I Cauchy slice is the semiclassical shadow of a Part III state. **Statuses.** Part I is **[S/H]**, Part III is **[S/H]**. **Composite.**

$$\mathrm{w}(S/H) \vee \mathrm{w}(S/H) = H \vee H = H.$$

So the composite is **[H]**, even though several internal theorems of both regimes (the slice theorem, the uniqueness of the kinematical measure) are pure S. The governing warrant of the *bridge* is H because the limit claim is only heuristically established, and theorem 2.6 correctly caps the composite there. This is the discipline of the calculus: composing two S/H regimes does not automatically give S/H; one composes the statuses of the entries actually used, and the bridge can be the weak link.

5.2 Part III with Part IV: kinematics plus dynamics

Claim. Spin networks (states) together with spin foams (amplitudes) give a complete covariant quantum-gravity dynamics. **Statuses.** **[S/H]** and **[H]**. **Composite.**

$$\mathrm{w}(S/H) \vee \mathrm{w}(H) = H \vee H = H.$$

The bare-H spin-foam refinement status is the weak link. This is why the roadmap's QG-III task demands that continuum-limit limitations be recorded explicitly rather than the kinematics-plus-dynamics package be asserted as settled.

5.3 Part VI with Part VII: residue tree equals coaction tree

Claim. For a double-copy-constructed gravity amplitude, the residue tree of the positive geometry coincides with the motivic coaction tree of the amplitude's period. This is open-problem 1 (section 10). **Statuses.** Part VI canonical-form mathematics **[S/H]**, Part VII double-copy mathematics **[S/H]**, and the coaction-tree conjecture **[H]**. **Composite.**

$$\mathrm{w}(S/H) \vee \mathrm{w}(S/H) \vee \mathrm{w}(H) = H \vee H \vee H = H.$$

Two S/H-solid constructions plus one heuristic conjecture give a composite capped at **[H]**. The computation is open-problem 1 restated as a status: the ingredients are secure, the bridge is conjectural, and the honest label follows.

5.4 Part II with Part XII: BV quantization on an indefinite causal background

Claim. A BV–BRST-quantized gravitational theory is placed on a background with genuinely indefinite (superposed) causal order. **Statuses.** Part II **[S/H]**, Part XII **[H/P]**. **Composite.**

$$\mathrm{w}(S/H) \vee \mathrm{w}(H/P) = H \vee P = P.$$

This is the sharpest illustration of theorem 2.9. However careful the BV–BRST construction is, chaining it to a claim that presupposes ontologically indefinite causal structure pulls the composite to **[P]**. A paper attempting this must present the result as speculative, not heuristic, regardless of the maturity of the gauge-theory half.

5.5 Part IX with Part IV: two ultraviolet completions

Claim. The asymptotic-safety fixed point and the spin-foam continuum limit describe the same ultraviolet completion. **Statuses.** **[H]** and **[H]**. **Composite.**

$$\mathrm{w}(H) \vee \mathrm{w}(H) = H.$$

The composite bound is **[H]**, but this is exactly the situation of theorem 2.10. No rigorous bridge between a continuum fixed point and a combinatorial refinement limit currently exists. The label H means only that neither endpoint exceeds heuristic status, so nothing built from them can. It does not assert that the bridge has been constructed. We list this as an open comparison in section 10.

5.6 All twelve at once

Claim. A single statement that depends on every regime simultaneously. **Composite.** By theorem 2.8,

$$\bigvee_{R\in\{\mathrm{I},\ldots,\mathrm{XII}\}} \mathrm{w}(\sigma_R) = \max\{S/H, \ldots, H/P\} = P,$$

driven entirely by Part XII's H/P. So any chain of reasoning spanning all twelve regimes is, at best, **[P]**. This is not a defect of the calculus; it is its designed behavior and the precise, checkable meaning of "modular, not unified." Many individual regimes (Parts I, II, III, VI, VII, X) are comfortably S/H on their own, but no argument that leans on all of them can be asserted above speculative status.

Remark 5.1. The compositions above are not exhaustive. Any path through fig. 1, or any subset of table 1, can be composed the same way. The only rule is theorem 2.6, and the only surprise the calculus ever produces is that a chain is weaker than a reader expected, never stronger.

6 Cross-cutting structural recurrences

Distinct regimes sometimes run on the same mathematical pattern. Recording these recurrences is useful because a lemma proved in one regime often transports, and because a shared pattern is a candidate for the formalization layer. We list the recurrences that thread more than one Part, being careful to mark them as structural analogies rather than identifications.

6.1 Labels as geometry

In Part III a spin-network edge label j_e is an $SU(2)$ irreducible representation, and the area of a surface is $\sum_e 8\pi\gamma\ell_P^2\sqrt{j_e(j_e+1)}$ over piercing edges. In Part XI the "label" is the spectrum of the Dirac operator D, and metric distance is $d(p,q) = \sup\{|a(p)-a(q)| : \|[D,a]\| \le 1\}$. Both regimes recover a geometric quantity (area, distance) from purely representation-theoretic or spectral input, with no metric posited in advance. The recurrence is real, but the two are separate reconstructions of Part I, not of each other (section 7).

6.2 Residue and coaction trees

Part VI builds an amplitude as the canonical form Ω_X of a positive geometry, whose defining data is a recursive tree of residues along boundary strata. Part VII builds gravity amplitudes from gauge-theory numerators, and the transcendental functions appearing carry a motivic coaction Δ with its own tree of coproducts $\Delta(\mathcal{A}^{\mathfrak{m}}) = \sum_i \mathcal{A}^{\mathfrak{m}}_{i,1} \otimes \mathcal{A}^{\mathfrak{m}}_{i,2}$. Open-problem 1 conjectures these trees agree. Part II's quantization-obstruction cohomology is a structurally analogous decomposition operation, and whether the two are literally adjoint (cut versus regulator) is open-problem 2. The recurrence here is a family of decomposition laws; the open problems are whether the family members coincide.

6.3 Gauge redundancy versus code redundancy

Part II resolves gauge redundancy: many field configurations, one cohomology class, Obs $= H^0(Q_{\mathrm{BV}})$. Part V resolves code redundancy: many boundary representatives, one bulk operator, $\mathcal{H}_{\mathrm{bdy}} \simeq (\mathcal{H}_{\log} \otimes \mathcal{H}_{\mathrm{gauge}}) \oplus \mathcal{H}_{\mathrm{err}}$. The two decompositions look alike, and the analogy between a gauge orbit and a code's logical subspace is suggestive. It is also only an analogy: open-problem 5 asks precisely which gauge redundancies behave like logical code degrees of freedom and which do not. We do not assert the identification; we record it as a structural parallel with a named open problem attached.

6.4 Local-to-global descent

Part II (BV–BRST observables), Part X (factorization algebras), and Part VIII (celestial data reconstructed from asymptotic regions) all assemble a global datum from local pieces by a descent or gluing operation. In Part X this is the defining axiom: $\mathrm{Obs}(U \cup V)$ is a composition, not a restriction. In Part II it is the statement that the BV observables form a factorization algebra whose quantization is a cohomological existence problem. In Part VIII it is the reconstruction of the S-matrix from data at null infinity. The shared pattern is the descent axiom of the representation stack (theorem 2.3); the regimes differ in what they glue.

6.5 Causal order

Causal order appears in three Parts with three different statuses. In Part I it is the classical light-cone structure of a Lorentzian metric, definite and **[S/H]**. In Part IV it is the combinatorial ordering of a spin-foam two-complex, **[H]**. In Part XII it is either a locally finite poset or a genuinely indefinite process matrix, **[H/P]**. The same word denotes progressively weaker-warranted objects as one moves from a classical metric to an operational process matrix, and the calculus records exactly that.

6.6 Emergent metric

A metric is an output in four regimes. Part I treats it as the classical datum to be recovered. Part XI recovers it from spectral data by the Connes distance formula. Part V recovers a bulk metric from boundary entanglement (the Ryu–Takayanagi area law). Part IX asks whether a continuum metric survives to the ultraviolet as an RG-invariant fixed-point structure. These are four independent routes to the same target, and their mutual consistency (that they all reproduce the Part I metric in the appropriate limit) is a requirement, not a theorem.

The last two rows of table 2 deserve emphasis. "Discreteness reconstructs continuum" names two independent programs—representation-theoretic (Part III) and order-theoretic (Part XII)—that make structurally parallel but technically unrelated claims about recovering Part I geometry from a discrete substrate. "RG as arbiter" names the same continuum question asked in two frameworks (Part IV refinement, Part IX fixed point) with no bridge between them. Both are honest gaps, not merely under-cited connections, and both reappear in section 10.

Recurrence	Parts	Nature
groupoid, not coarse quotient	I, II, III	isotropy/stabilizer retained
labels as geometry	III, XI	geometry from rep/spectral labels
residue / coaction trees	VI, VII (, II)	decomposition laws; open-problems 1–2
gauge vs. code redundancy	II, V	analogy; open-problem 5
local-to-global descent	II, X, VIII	gluing/composition axiom
causal order	I, IV, XII	same word, weaker warrants
emergent metric	I, XI, V, IX	four routes to one target
discreteness $\to$ continuum	III, XII	rep-theoretic vs. order-theoretic
RG/coarse-graining as arbiter	IV, IX	refinement vs. fixed point (unbridged)

Table 2: Structural recurrences that thread more than one regime. Each is a shared pattern, not an identification; the ones with named open problems are flagged.

7 Explicit non-identifications

The modular discipline is as much about what is kept apart as about what composes. The following identifications are *not* licensed, and a downstream argument must not assert any of them without an explicit new construction and an explicit status recomputation by theorem 2.6.

1. **Celestial holography is not anti–de Sitter holography.** Part V (tensor-network/QEC, anti–de Sitter asymptotics, entanglement wedges and Ryu–Takayanagi surfaces) and Part VIII (celestial, flat asymptotics, Mellin transforms and BMS Ward identities) are complementary but structurally different realizations of "holography." They rest on different invariants over different bulk asymptotics. Theorem 2.6 should not be invoked to merge them into a single dictionary entry; they are presented side by side, not identified.

2. **Spin-network geometry is not spectral-triple geometry.** Parts III/IV (representation-theoretic quantum geometry) and Part XI (spectral noncommutative geometry) are separate candidate reconstructions of Part I, not two descriptions of one another. No equivalence between a spin network and a spectral triple is established, and the dashed arrows of fig. 1 deliberately do not connect them.

3. **The asymptotic-safety fixed point is not the spin-foam continuum limit.** Absent an explicit comparison, Part IX's fixed point and Part IV's refinement limit

must not be asserted to describe the same ultraviolet completion. As computed in section 5, the calculus bounds a claim linking them at **[H]**, but the bridge itself is unbuilt. This is an open comparison, flagged again in section 10.

4. **Mathematical consistency of indefinite causal order is not its gravitational realization.** Part XII's process matrices are mathematically consistent, and some violate causal inequalities. This does not establish that quantum gravity dynamically produces indefinite causal order. The step from consistency to realization is a motivational analogy, and the P component of Part XII's status marks exactly that step.

These non-identifications are why the paper's title says "site," not "theory." A site is a collection of local pieces with specified overlaps and specified non-overlaps. The overlaps are section 6; the non-overlaps are here.

8 The module map

The twelve regimes realize the roadmap modules QG-I through QG-VIII, plus two proposed extensions QG-IX and QG-X for the two regimes not named in the original eight-module roadmap. Table 3 records the map with composite statuses.

Two entries in table 3 warrant a note. QG-VIII, the formalization layer, does not have a fixed status: it inherits the status of whatever claim is being formalized. What is S there is the calculus itself—theorems 2.7 to 2.9 are ordinary lattice theory and are as secure as their proofs. The proposed QG-IX and QG-X are the program's own suggestion, not part of the inherited roadmap; they close the gap left by Parts XI and XII, which carry dictionary entries but no module number.

9 Emergence from composition

A recurring feature of the ladder is that composing two regimes produces structure present in neither factor. We isolate the theme here because it is what makes the modular site more than a filing cabinet, and because it is disciplined by the same calculus: emergent structure does not raise the composite status.

- **Level 0 with Level 1 (Parts I, II).** The ghost-for-ghost (reducibility) tower of the gravitational BV–BRST complex is emergent. It is invisible in the coarse moduli space of Part I and invisible in a naive BRST complex; it appears only when the derived critical locus records the isotropy of Part I as antighost data. The tower is nontrivial exactly in the degrees dual to $\mathrm{Lie}(\mathrm{Isom}(M, g))$, so the classical Killing isotropy reappears as cohomological structure one level up. Status of the composite: **[S/H]**, unchanged.
- **Level 2 with Level 3 (Parts III, IV).** The Regge-action asymptotics of the spin-foam vertex are emergent. A single $\{15j\}$ symbol is a piece of representation-theoretic combinatorics with no geometry in it; its large-spin asymptotics reconstruct the cosine of a Regge action, so curved simplicial geometry emerges from a sum over labels. Status of the composite: **[H]**, capped by Part IV.

Module	Regimes realizing it (Parts)	Composite status
QG-I	I (Lorentzian geometry)	**[S/H]**
QG-II	II (BV–BRST), X (factorization algebras)	**[S/H]** (local/perturbative)
QG-III	III (spin networks), IV (spin foams)	**[H]**
QG-IV	VI (positive geometries), VII (double copy)	**[S/H]** $\to$ **[H]** with conj. 1
QG-V	V (tensor networks / QEC)	**[H]**
QG-VI	VIII (celestial CFT)	**[H]**
QG-VII	IX (asymptotic safety)	**[H]**
QG-VIII	formalization (cross-cutting)	tracks the formalized claim; calculus itself **[S]**
QG-IX	XI (noncommutative geometry) *[proposed]*	**[H]**
QG-X	XII (quantum causal structures) *[proposed]*	**[H/P]**

Table 3: The twelve regimes mapped to modules QG-I–QG-X. QG-IX and QG-X are extensions proposed by this program for the two regimes (Parts XI, XII) not named in the original QG-I–QG-VIII roadmap; they are presented as natural extensions, not as already-adopted modules.

- **Level 3 internal (Parts VI, VII).** The transcendental-weight structure of a gravity amplitude is emergent from the gauge-squared structure. Neither a single gauge amplitude nor the bare double-copy replacement exhibits the full coaction; it appears in the composite, and (conjecturally) matches the residue tree of the positive geometry. Status: **[S/H]**, degrading to **[H]** with the conjecture.

- **Level 4 with Level 1 (Parts V, II).** The identification of complementary recovery with gauge invariance is a candidate emergent structure: the commutant/subalgebra structure of the code (Part V) looks like the gauge-invariant observable algebra of Part II. It is emergent in the sense that it appears only when the two are laid on top of each other, and it is only partial, which is open-problem 5. Status: **[H]**, capped by Part V.

- **Transverse (Parts XI, I; XII, I).** An emergent metric (Part XI, via the distance formula) and an emergent causal manifold (Part XII, via order plus counting) are structures that appear only when the algebraic or order-theoretic data of the transverse regimes is pushed to the Level-0 target. Neither the algebra alone nor the poset alone is a metric; the metric emerges as a reconstruction theorem or conjecture. Status: **[H]** and **[H/P]** respectively.

In every case the emergent structure is genuine—it is not present in either factor in isolation—and in every case the composite status is the join of the factors, not better. Emergence and epistemic honesty coexist because theorem 2.8 governs the label while the mathematics governs the content.

10 Limitations and open problems

10.1 Limitations of this synthesis

This paper is a map, and a map is not the territory. Three limitations are worth stating plainly.

First, the warrant calculus is a bookkeeping device, not a physical dynamics. It tells the reader how secure a composite claim is; it does not tell the reader whether the composite claim is true, or whether the bridge it names has ever been constructed (theorem 2.10). A composite status of H is a ceiling, not a floor.

Second, the twelve regimes are unequal in maturity, and the synthesis does not hide this. Parts I, II, III, VI, VII, and X have substantial S content; Parts IV, V, VIII, IX, XI carry bare H; Part XII is proposed at H/P. Any impression of a level playing field would be false, and the status column of table 1 is the antidote.

Third, the module extensions QG-IX and QG-X are this program's proposals. They are natural, they close a real gap, and they are not yet part of the inherited roadmap. We have marked them as proposed throughout.

10.2 The consolidated open-problem queue

The following problems are drawn from the roadmap and from the cross-cutting analysis above. They are the concrete work items the modular site generates.

1. **Residue tree equals coaction tree.** Prove or refute, in explicit amplitude and positive-geometry examples, that the residue tree of Part VI coincides with the motivic coaction tree of the period. (Section 5; Parts VI, VII.)

2. **Cut/regulator complementarity.** Formalize whether discontinuity/cut maps are adjoint to regulator/period maps in a suitable category, connecting the decomposition operations of Parts VI–VII to the quantization-obstruction cohomology of Part II. (Section 6.)

3. **Elliptic and non-Tate extension.** Extend the coaction machinery beyond mixed Tate motives to elliptic and modular periods, rather than assuming the mixed Tate case covers all amplitudes. (Part VI.)

4. **Stack of gravitational solutions.** Construct the diffeomorphism quotient and its isotropy as a stack for concrete solution families (Schwarzschild, Kerr, FLRW), discharging theorem 2.4 in named cases rather than schematically. (Part I, and its perturbative-quantum version in Part II.)

5. **Holographic-QEC bridge.** Determine which gauge redundancies behave like logical code degrees of freedom and which do not, making the Part II/Part V analogy of section 6 precise. (Parts II, V.)

6. **Celestial dictionary entries.** Encode the assumptions behind scattering basis transforms, soft sectors, and asymptotic-symmetry constraints, keeping the Part VIII status separate from anti–de Sitter holography. (Part VIII; section 7.)

7. **Formalization.** Turn the status calculus, the descent checks, and the small geometry/coalgebra lemmas into machine-checkable modules, starting from theorems 2.7 to 2.9, which are already in a form a proof assistant can accept. (QG-VIII; all Parts.)

8. **Two ultraviolet completions.** Compare the spin-foam continuum limit (Part IV) and the asymptotic-safety fixed point (Part IX) directly. As of this writing no literature bridges the two, and section 5 shows the calculus can only bound, not build, such a bridge. This is a genuine gap, not an under-cited connection. (Parts IV, IX; section 7.)

9. **Two discretization programs.** Compare the representation-theoretic (Part III) and order-theoretic (Part XII) reconstructions of Part I geometry systematically. Their parallelism is structural, not established, and no systematic comparison exists in the literature. (Parts III, XII; section 6.)

Items 1–7 are the roadmap's own queue. Items 8–9 are cross-topic gaps surfaced by this synthesis and not present as such in the roadmap; we flag them as the synthesis's own contribution to the problem list.

11 Conclusion

Twelve regimes, one question. The question is the reconstruction thesis: spacetime, matter, causality, and measurement as outputs of invariant representation structures. The twelve answers are a classical solution stack, a gauge cohomology, a spin-network Hilbert space, a state-sum amplitude, a holographic code, a canonical form, a double-copy numerator, a celestial correlator, a renormalization-group fixed point, a factorization algebra, a spectral triple, and a process matrix. Each reconstructs some part of the target, and each carries a warrant that says how securely.

What holds them together is not a master theory but a discipline. The warrant lattice $\mathsf{W} = \{S < H < P\}$ and its join composition make "modular, not unified" into a proved statement: a chain is no stronger than its weakest link (theorem 2.8), speculative links are never laundered (theorem 2.9), and a claim spanning all twelve regimes is capped at speculative status by the single H/P regime it must pass through. Inside that discipline the regimes do real work together: composing them produces emergent structure at every level (section 9), and shared patterns transport lemmas across regimes (section 6). Outside it, the non-identifications (section 7) keep celestial and anti–de Sitter holography apart and leave the two ultraviolet-completion questions unbridged.

The honest summary is that this is a map of a site, drawn to scale, with the secure regions and the speculative regions colored differently and the roads that do not yet connect marked as such. The open-problem queue of section 10 is the list of roads worth building. Building them is the next phase; drawing the map accurately is this one.

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